A NONCONTRADICTORY METHOD FOR CALCULATION OF RADIANT TRANSFER AND THE QUESTION OF SHOCK WAVE STRUCTURE

A. I. Vyskrebentsev, V. A. Nuzhnyi, and Yu. P. Raizer UDC 533.6.011.72

1. Introduction. The structure of the front of very intense shock waves and the intensity of the radiation produced by the front surface are determined by radiant heat exchange in the front region [1]. The radiation of the hot gas, exiting from behind the compression discontinuity, heats gas which has not yet been shock-compressed. The temperature ahead of the discontinuity itself T_- rapidly increases with increase in wave velocity D and temperature behind the front T_f. At some critical value $D = D_c$ the temperature T_- reaches the value $T_f (T_- = T_f = T_c)$, then with further increase in amplitude ($D > D_c$, $T_f > T_c$) remains equal to T_f, while an ever lengthening "tongue" of gas heated by radiant heat transfer is formed ahead of the discontinuity. Qualitative analysis of the equations of [2] or their approximate analytical solution [3] leads to this pattern, and the dependence of front luminosity on amplitude calculated on this basis [4] agrees with observation results.

At Los Alamos Laboratory in 1973 Zinn and Anderson [5], following [1-4], performed a numerical calculation of the radiant transfer and gasdynamics equations for a steady-state shock wave in air. The goal of that study was apparently obtaining refined and reliable quantitative data, unachievable by the simplified analytical solution of [3, 4] (we note that the major mathematical results of [3, 4] were confirmed by the calculations of [5] with accuracy beyond that expected). However, one significant result obtained in [5] proved quite strange. For waves with amplitude close to critical, the iteration process did not converge to a finite solution. The highest wave amplitude for which a solution could still be obtained corresponded to D = 80 km/sec, T_f = 258,000°K, T_- = 244,000°K, (incidentally, this was confirmed well by the estimates of [3]: $T_f \doteq T_- = T_c$ = 285,000°K at $D = D_c$ = 88 km/sec).

From this fact the authors concluded that, in general, supercritical steady-state shock waves do not exist. A wave in air with D > 80 km/sec is transient, and is in fact converted into a thermal wave [1], driven by radiant thermal conductivity. In this case the front velocity D is an eigenvalue of the steady-state regime equations for a definite pair of T and dT/dx values (i.e., energy flux) behind the front; the role of hydrodynamics is then negligible. There is no unique final state with dT/dx = 0 corresponding to a steady-state shock wave. The authors also noted that they did not obtain such a solution with flow behind the front.

A conclusion of such a radical nature will perhaps not drastically change our concepts of the situation which develops upon a strong explosion in air, where the transition from thermal to shock wave most often occurs at D ~ 90 km/sec and T ~ 300,000°K [1], but it is certainly of principal significance with respect to shock-wave theory. One can easily conceive of situations in which a steady-state shock wave would exist with a velocity higher than 80-100 km/sec, for example, if a "piston" moving at appropriate velocity or an adequate pressure exists behind the front.

The conflict in the theory must be resolved, and for this purpose a numerical solution of the problem was undertaken using formulation close to that of [5] in all expressions of principal significance, but also simplified as much as possible with regard to details which could not lead to qualitative consequences. In evaluating the results of the calculation, we will note immediately that they confirmed the conclusions of the theory of [1-4] as to the existence of shock waves of supercritical amplitude. We will demonstrate the most probable error in the computation technique used in [5], which leads to an incorrect result, and should be kept in mind when performing numerical solutions of radiant gas dynamics problems in the future.

Moscow. Translated from Zhurnal Prikladnoi Mekhaniki i Tekhnicheskoi Fiziki, No. 6, pp. 42-46, November-December, 1983. Original article submitted October 21, 1982.

 0021-8944/83/2406-0802$07.50

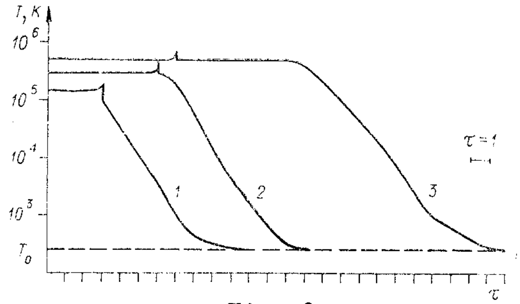

Fig. 2

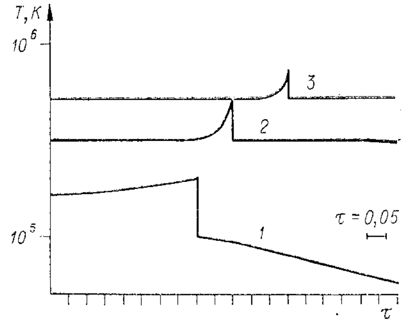

Fig. 3

In [6], which considered the problem of shcok wave front structure with consideration of radiation, the calculations were performed for sufficiently low amplitudes, in which case the heating temperature ahead of the front T_- does not reach the temperature behind the front T_f.

2. Formulation of the Problem and Equations. The general formulation of the problem of shock-wave structure with consideration of radiant heat exchange in the present study will be the same as that of [1-5].

We will consider a one-dimensional steady-state regime in a coordinate system fixed to the wave front (Fig. 1). The x axis is directed in the direction of the flow. The gas velocity u is positive, and the radiant flux density S, negative. Denoting the pressure, density, and specific internal energy by p, ρ, ε, we write the integrals of the continuity, motion, and energy equations in the form

$$\rho u = \rho_0 u_0; \tag{2.1}$$

$$p + \rho u^2 = p_0 + \rho_0 u_0^2; \tag{2.2}$$

$$\rho u\left(\varepsilon + \frac{p}{\rho} + \frac{u^2}{2}\right) + S = \rho_0 u_0\left(\varepsilon_0 + \frac{p_0}{\rho_0} + \frac{u_0^2}{2}\right). \tag{2.3}$$

The subscript 0 denotes values in the unperturbed gas before the wave front at $x = -\infty$. We neglect the radiation flux departing from the wave into the cold gas, i.e., $S(-\infty) = 0$; $u_0 \simeq D$, where D is the velocity of shock wave motion through the cold gas.

All quantities on the left of the equation are functions of coordinate x. The gas thermodynamic properties $\varepsilon(p, \rho)$, $p(\rho, T)$ are assumed known. Behind the front at $x = +\infty$ the temperature reaches a value constant along x, T_f, and the radiation flux disappears: $S(+\infty) = 0$.

To solve the problem put forward herein, a very simple variant of radiant transfer calculation was chosen. To describe the angular distribution of intensity, we will use the forward–back approximation of [1], in which unidirectional fluxes traveling in positive S^+, and negative S^-, directions are introduced (Fig. 1). Then

$$S = S^+ - S^-. \tag{2.4}$$

We will operate with the integral radiant flux over the spectrum and the corresponding mean (over spectrum) absorption coefficient, corrected for constrained radiation ϰ although this is not of principal import. In fact, in place of the coordinate x we introduce the optical thickness

$$\tau = \int_{-\infty}^{x} \varkappa dx,$$

measuring this quantity from the unperturbed gas. We will now seek distributions of the temperature and other quantities over τ rather than x.

As in [5], we divide the coordinate scale into segments — plane layers of optical thickness h_i (not necessarily identical). We assign values on the boundaries of layer i the subscripts i and i + 1.

Equations of the form of Eqs. (2.1)-(2.3) relate the quantities on the boundaries of any layer:

$$\rho_i u_i = \rho_{i-1} u_{i-1}; \tag{2.5}$$

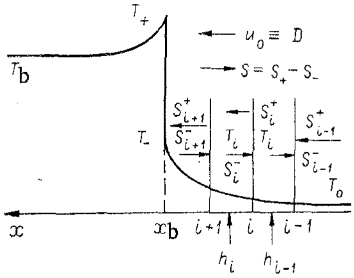

Fig. 1

$$p_i + \rho_i u_i^2 = p_{i-1} + \rho_{i-1} u_{i-1}^2; \quad (2.6)$$

$$\rho_i u_i \left(\varepsilon_i + \frac{p_i}{\rho_i} + \frac{u_i^2}{2} \right) + S_i = \rho_{i-1} u_{i-1} \left(\varepsilon_{i-1} + \frac{p_{i-1}}{\rho_{i-1}} + \frac{u_{i-1}^2}{2} \right) + S_{i-1}. \quad (2.7)$$

To calculate the unidirectional fluxes at the i-th boundary it is necessary to consider two adjacent layers; we then assume the temperature within the layers from i — loto i + 1 to be the same and denote them by the subscript i. In the forward—back approximation the following relationships between the unidirectional fluxes follow from the transfer equations (see Fig. 1):

$$S_i^+ = S_{i-1}^+ e^{-2h_{i-1}} + \sigma T_i^4 (1 - e^{-2h_{i-1}}); \quad (2.8)$$

$$S_i^- = S_{i+1}^- e^{-2h_i} + \sigma T_i^4 (1 - e^{-2h_i}), \quad (2.9)$$

where σ is the Stefan—Boltzmann constant.

The meaning of these expressions is quite simple: The first terms are the fluxes flowing out through the adjacent boundary and attenuated in the given layer by absorption, while the second terms are the intrinsic radiation of the layer with consideration of self-absorption; the factor of two appears in the exponent because of lengthening of the paths of oblique rays. In principle, Eqs. (2.8), (2.9) are equivalent to the analogous computation expressions of [5], but simpler because of the very coarse manner in which oblique rays are considered, integral exponents being replaced by ordinary ones.

We assign conventional forms to the thermodynamic functions:

$$\varepsilon = [1/(\gamma - 1)]p/\rho, \quad T = \varepsilon/c_V, \quad (2.10)$$

but will consider the adiabatic index γ and specific heat c_V to be dependent on temperature (and density). Corresponding values are found with the aid of tables [7]. Without considering the temperature dependence of specific heat, and in part, that of adiabatic index, it is impossible to obtain a correct relationship between temperature behind the front and shock-wave front velocity; the divergence from the real function $T_f(D)$ in air is so great that the solution becomes quite abstract.

The boundary conditions for Eqs. (2.8), (2.9) are as follows: at $x = -\infty$ $S^- = \sigma T_0^4$, where T_0 is the temperature of the cold gas, which for convenience in calculation we will consider nonzero; behind the front at $x = +\infty$ $S^+ = S^-$, since the temperature asymptotically tends to $T(x) = \text{const} = T_f$.

The last condition is equivalent to $S^-(+\infty) = \sigma T_f^4$.

3. Numerical Calculation Method. The structure equations were solved by the iteration method. However, the simplest possible iteration process, consisting of calculating the radiant fluxes from the temperature distributions obtained in the previous iteration, produces convergent solutions only for waves of relatively low (precritical) amplitude.

At high T_f not even the first iteration can be performed. The flux ahead of the discontinuity itself in the zero-th approximation is σT_f^4 and is so great that the temperature ahead of the discontinuity, equal to [1]

$$T_- = \frac{\sigma T_b^4}{\rho_0 c_V (T_-) D}, \quad (3.1)$$

proves to be greater than T_f. For example, at D = 100 km/sec, $T_f = 4 \cdot 10^5$ °K, and from Eq. (3.1) we obtain $T_- = 8.7 \cdot 10^5$ °K, more than double T_f. Even without considering that this violates the second law of thermodynamics [1], the flux S obtained formally in the next ap-

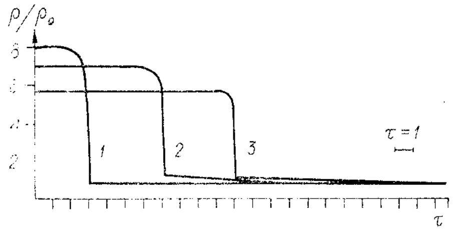

Fig. 4

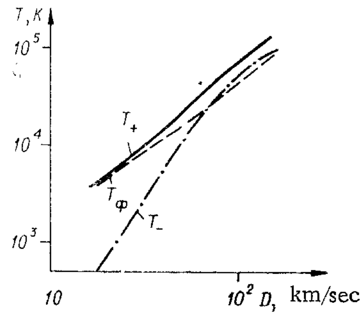

Fig. 5

proximation from such a temperature distribution travels in the direction from the cold gas into the hot, which is physically meaningless.

This contradiction was noted in [8], but to avoid difficulties an empirical method was proposed, based on reducing the flux by a specific factor, and repeating this process at each iteration until the contradiction was eliminated. Such a procedure requires solution of the equations for a series of reduced fluxes with no correspondence to the physical essence of the problem, and is inconvenient since the factor by which the flux must be reduced is not known beforehand.

The principle of the method to be present below is to eliminate the possibility of contradiction itself, by matching the calculation procedure to those factors which in fact do not permit the temperature at any point ahead of the discontinuity to become higher than the temperature behind the front. The natural radiation of the absorbing layer limits the temperature. Thus, we isolate from the flux equations for the fluxes the term proportional to σT^4, which is then introduced into the balance equations (2.5)-(2.7), so that it is considered in determining the temperature distribution in each iteration.

The calculation procedure consists of the following. The flux difference $S_{i+1} - S_i$ is expressed in the form $(dS/d\tau)_i h_{i-1}$, where $dS/d\tau$ at the point τ_i is found by differentiating Eqs. (2.4), (2.8), (2.9), with the assumption that in the interval from τ_{i-1} to τ_{i+1} $T = \text{const} = T_i$. By eliminating ρ_i, p_i, u_i the systems (2.5)-(2.7), (2.10) are reduced to the form

$$\left(1+\frac{\gamma_i-1}{4}y_i\right)\left[\frac{\gamma_i-1}{4}y_i+\sqrt{\left(1-\frac{\gamma_i-1}{4}y_i\right)^2-y_{i-1}(\gamma_{i-1}-1)-1}\right]+\frac{\gamma_i}{2}y_{i-1}-\frac{\gamma_i+1}{2}y_i+a_i=0, \quad (3.2)$$

where $y_i=\frac{4\varepsilon_i}{u_{i-1}^2}$; $a_i=-\frac{4}{\rho_{i-1}u_{i-1}^3}\left(\frac{dS}{d\tau}\right)_i h_{i-1}$ and the "flux difference" a_i satisfies the condition

$$a_i=\alpha_i y_i^4-\beta_i,\quad \alpha_i=\frac{\sigma h_{i-1}u_{i-1}^5}{2\rho_{i-1}c_{Vi}}[e^{-2h_{i-1}}+e^{-2h_i}],\quad \beta_i=\frac{8h_i}{\rho_{i-1}u_{i-1}^3}[S_{i+1}^-e^{-2h_i}+S_{i-1}^+e^{-2h_{i-1}}]. \quad (3.3)$$

If the fluxes a_i are found from a temperature distribution produced by the quantity y_{i-1}, i.e., if the quantity a_i is assumed temperature-independent in calculating the last expression of Eq. (3.2), then this equation would be quadratic in y_i. We will introduce the term $a_i y_i^4$, figuring in a_i [Eq. (3.3)] and describing the natural radiation of the layer, and in Eq. (3.2) in the calculation of y_i. Thus, the equation will be fourth-order. This complication is compensated by the fact that we no longer need be concerned with the appearance of contradiction. In accordance with the actual physics of the problem, the temperature does not increase above the value dictated by the combined action of light emission and absorption. At very high wave amplitudes Eq. (3.2) is satisfied by a root value which is close to the solution of the equation $a_i(y_i) = 0$, corresponding to compensation of unidirectional fluxes. It is evident from Eq. (3.2) that the temperature cannot then be higher than the effective temperature of the radiation transferred by the fluxes S_{i+1}^- and S_{i-1}^+. Even in the first iteration T_- cannot exceed T_f, as must be in supercritical waves. The calculations performed revealed that the simple iteration method without separation of the layer natural radiation is unusable for calculation of supercritical amplitude shock waves: The first iteration procedure caused a machine crash. Apparently this was the cause of the unsuccessful calculations of [5] for waves with D > 80 m/sec.

We will note [9, 10], in which natural radiation was considered in determining nonequilibrium factors to increase the stability and convergence of the iterations in solving the energy equations and nonstationary gasdynamics problems.

4. Calculation Results and Evaluation. Results of calculations for shock waves in normal density air are presented in Figs. 2-5 (T_0 = 300°K).

Figure 2 shows distributions of temperature T over optical thickness τ. Waves of three amplitudes were calculated: one of precritical amplitude with D - 55.6 km/sec (curves 1 in Figs. 2-4), and two supercritical with D = 85.9 km/sec (curves 2) and D = 110 km/sec (curves 3). The distribution for the various waves have been shifted relative to each other by an arbitrary distance. This was done only for the reader's convenience in examining the graphs. Figure 3 shows the same distributions as Fig. 2, but in large scale for the region near the compression discontinuity, in order to show details of the temperature peak. Figure 4 presents density distributions, or more accurately, the degree of compression, ρ/ρ_0, where ρ_0 = $1.29 \cdot 10^{-3}$ g/cm^3. Figure 5 shows temperatures behind the front T_f, ahead of the compression discontinuity T_-, and behind the discontinuity T_+ as functions of front velocity D.

The temperature distributions are in complete qualitative agreement with predictions of analytical theory, so the question of the possibility of existence of supercritical waves is answered unambiguously and positively.

Generally speaking, it would be possible to transform from distributions over τ to distributions over spatial coordinate, if we choose a function $\varkappa$(T) in a reasonable manner. However, there is no special need to do this. For subcritical waves the calculations of [5] are correct. These provide high accuracy because of the detailed consideration of radiation spectral characteristics. For supercritical waves $\varkappa(T)$ is essentially characterized by the Rosseland path — the distribution was discussed in [1, 3].

We will now offer a few words concerning realization of shock waves with supercritical amplitude. A shock wave must be produced, if the conditions, in particular, boundary conditions, are favorable. For example, if behind the wave there acts a "piston" which drives the gas at high velocity, exceeding the supercritical value, then a shock wave, and not a thermal wave, will be produced.* Supercritical amplitude waves may be obtained in explosions of sufficient intensity, and also in a gas of reduced density, where the critical transition is accomplished at lower temperatures behind the front, i.e., for weaker waves.

LITERATURE CITED

1. Ya. B. Zel'dovich and Yu. P. Raizer, Physics of Shock Waves and High Pressure Hydrodynamic Phenomena, Academic Press.
2. Ya. B. Zel'dovich, "High amplitude shock waves in gases," Zh. Eksp. Teor. Fiz., 32, 1126 (1957).
3. Yu. P. Raizer, "The structure of a strong shock wave front in a gas," Zh. Eksp. Teor. Fiz., 32, 1528 (1957).
4. Yu. P. Raizer, "Luminosity of strong shock wave fronts," Zh. Eksp. Teor. Fiz., 33, 101 (1957).
5. J. Zinn and R. C. Anderson, "Structure and luminosity of strong shock waves in air," Phys. Fluids, 16, No. 10 (1973).
6. V. V. Aleksandrov and V. N. Koterov, Use of the Spherical Harmonic Method in Study of Shock Wave Structure in a Gray Radiating Gas [in Russian], VTs Akad. Nauk SSSR (1974).
7. N. M. Kuznetsov, Thermodynamic Functions and Shock Adiabats of Air at High Temperatures [in Russian], Mashinostroenie, Moscow (1965).
8. R. Goulard and H. F. Nelson, "Numerical convergence of highly radiating shock waves — Chapin's multiplying factor," in: Proceedings of the Section on Numerical Methods in Gas Dynamics, Second International Colloquium on Gas Dynamics of Explosion and Reacting Systems [in Russian], VTs Akad. Nauk SSSR, Moscow (1971).
9. V. Ya. Gol'din and B. N. Chetverushkin, Methods for Calculation of Radiant Transfer in One-Dimensional Low-Temperature Plasma Problems [in Russian], Preprint No. 2, Inst. Prikl. Mekh., Akad. Nauk SSSR (1970).
10. B. N. Chetverushkin, "A method for simultaneous solution of the energy and radiant transfer equations," Zh. Vychisl. Mat. Mat. Fiz., 10, 1290 (1970).
11. I. V. Nemchinov, V. V. Svettsov, and V. V. Shuvalov, "Structure of the heated layer ahead of the front of a high-power intensely radiating shock wave," Zh. Prikl. Mekh. Tekh. Fiz., No. 5 (1978).

*We note that [11] solved the problem of radiant heat exchange in a wave driven by a piston for subcritical amplitude.